Some Expressions of the Variety of Forest and its Influence on the Productivity of Labor of the Felling Operation

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Contents

I. Introduction

We know that the variation or the condition of the forest itself has a lot of influence on the productivity of labor of the work of forestry.

The subject of this thesis is to investigate the relation of the working rate of the felling operation to the variety of the forest which is one of the field conditions.

The variety of the forest, worker's character, machines and tools they use, weather, and etc. are the conditions which have some influence on the producivity of the felling operation, and among them the variety of the forest is one of the most remarkable. The researchers who conduct some experiments about some operations of the logging in field are sometimes strongly required to have enough knowledge about the influence of the field condition, because it is very difficult to randomize the bias in the data caused by it. To know what influence it has on the working rate of felling, we have to know first of all what the field condition is and how it should be grasped and expressed. So, in the first place, the variety of the place of felling must be analyzed, understood, and represented. But it could not be so easy to know all of it or what it is due to Anyhow it is very difficult to do perfect analysis of the variety of the place of felling.

We can utilize two mathematical methods in statistics for that purpose. One is so called component analysis, by which we can arrange so complicated appearance of the forest in proper shape; and the other is so called canonical analysis, which is in a sense a technique to analyze the variety in relation to some external object.

In the first half of the thesis, it is tried to analyze and express the variety of the place making use of those methods, and then the latter half describes the influence of it on the working rate of felling.

This thesis has been developed in support of Professor H. Sugihara. Acknowledgements are also due to other members of the department of forestry of Kyoto University T. Shidei and R. Endo. Almost all of the computations were performed by KDC-I, an electronic computer, in the computation center of Kyoto University.

II. The variety of the place of felling and its expression.

1. Basal measures

As one of the most primitive and intuitive ways to represent the appearance of the felling place, we can think of the one by directly measurable factors such as species of trees standing there, mean D.B.H. of them, density, gradient and so on. Those factors will be called "basal measures" for convenience' sake hereafter.

It may be sometimes conceived that it is convenient to use such basal measures to study about the external appearance of the place of felling, but in fact that is not so. Such basal measures must be thought to have some correlations among themselves inevitably. We sometimes find difficulty in analyzing the variety of the place of felling because of that property.

It is easy to propose such an instance. Here is the one which was observed in Kyoto University Forest at Ashu in 1961. The exact place of the investigation was a part of the 16th block of Ashu Forest, which was the natural forest where Sugi (Cryptomeria) and some species of broad leaved trees (Beech, Japanese Oak, etc.) mixed. And there only Sugi was cut. Twenty-four plots were set for the investigation at random over all the felling area (about 12 hectares). Each plot was about 200—400 square-meters in area and included ten cut trees (Sugi). At each we measured nine basals as follows: mean gradient, density of all standing timber in number, density of cut trees in number, mean D.B.H., variance of D.B.H., mean height, density of cut trees in volume, mean clear height, and mean number of branches. The last seven factors were measured only about cut trees. Here, "mean" of "mean D.B.H.", "mean height", etc. are meaning "average in each plot".

The correlation matrix of those nine basal measures is on Table II. 1.

Looking at the correlation matrix, it is understood that the correlations between mean D.B.H. and mean height, or between gradient and density of cut trees in volume, etc. were so large that we can not neglect them as occured by chance. Consequently, a change in one factor, for instance, mean D.B.H. must be considered to bring about inevitably some change in all other factors correlating with it, and on the contrary any other factor's change occasions some change in mean D.B.H.. Therefore, a factor, for instance, mean D.B.H. has to be considered to represent not only an aspect named mean D.B.H., but a mixture of all other

Table. II. 1. Correlation matrix of basal measures (Number of data: 24)

	1. Mean gradient	2. Density of all standing timber in number	3. Density of cut trees in number	4. Mean D.B.H.	5. Variance of D.B.H.	6. Mean height	7. Density of cut trees in volume	8. Mean clear height	9. Mean number of branches
1. Mean gradient	−1.0000	−0.3081	−0.5218	−0.1110	0.2567	0.0119	−0.5290	−0.4424	0.1709
2. Density of all standing timber in number	−0.3081	1.0000	0.6069	−0.3301	−0.5457	−0.2322	0.2973	0.0514	0.1364
3. Density of cut trees in number	−0.5218	0.6069	1.0000	−0.0414	−0.4425	−0.1187	0.7851	0.2766	−0.1064
4. Mean D. B. H.	−0.1110	−0.3301	−0.0414	1.0000	0.3926	0.6669	0.5344	0.1951	0.3515
5. Variance of D. B. H.	0.2567	−0.5457	−0.4425	0.3926	1.0000	0.1251	−0.1338	−0.0695	0.3778
6. Mean height	0.0119	−0.2322	−0.1187	0.6669	0.1251	1.0000	0.3779	0.1667	0.4556
7. Density of cut trees in volume	−0.5290	0.2973	0.7851	0.5344	−0.1338	0.3779	1.0000	0.3510	0.1019
8. Mean clear height	−0.4424	0.0514	0.2766	0.1951	−0.0695	0.1667	0.3510	1.0000	−0.4603
9. Mean number of branches	0.1709	−0.1364	−0.1064	0.3515	0.3778	0.4556	0.1019	−0.4603	1.0000

factors implicitly, So ultimately we can not know exactly what it is really representing.

Thus the basal measures, having some correlation among themselves, are neither easy to be dealt with nor convenient as the ground for discussion. To avoid that property, it is necessary to bring such factors as are statistically independent or as do not have any correlation with one another.

There is a conception of principal components in statistics which satisfies that request.

2. Characteristic components

The author wants to call those principal components "characteristic components", because, as explained later on, those can be regarded as expressing conceptual characteristics of the felling place such as the shape of tree, the type of forest and so on.

2.1 Some properties of "principal components"

There are many literatures on component analysis which is a way to compute principal components. But it will be convenient to explain here some properties of principal components for later discussion.

Now suppose we have values of p basal measures observed on n plots. We write X_{ij} for the jth observation on the ith basal measure so that the values may be arrayed in a matrix:

$$\begin{pmatrix} X_{11}, & X_{12}, & \cdots\cdots X_{1n} \\ X_{21}, & X_{22}, & \cdots\cdots X_{2n} \\ X_{31}, & X_{32}, & \cdots\cdots X_{3n} \\ - & - & \cdots\cdots \\ X_{p1}, & X_{p2}, & \cdots\cdots X_{pn} \end{pmatrix} \qquad \cdots\cdots\cdots (II.1)$$

and standardizing them

$$\begin{matrix} x_{11}, & x_{12}, & \cdots\cdots\cdots x_{1n} \\ x_{21}, & x_{22}, & \cdots\cdots\cdots x_{2n} \\ - & - & \cdots\cdots\cdots \\ - & - & \cdots\cdots\cdots \\ x_{p1}, & x_{p2}, & \cdots\cdots\cdots x_{pn} \end{matrix} \quad \cdots\cdots\cdots(\text{II. }2)$$

Here standardizing is to even all scales by transforming variables as follows:

$$x_{ij}=\frac{X_{ij}-X_{i}\cdot}{s(X_i)}$$

provided

$$X_i\cdot=\frac{1}{n}\sum_j X_{ij} \quad \text{and} \quad s(X_i)=\frac{1}{n}\sum_j(X_{ij}-X_i\cdot)^2.$$

If the original values X_{ij} are normally distributed, the standardized values x_{ij} are subject to the standard normal distribution N(0, 1).

Then we get the principal components $\zeta_\alpha(\alpha=1, 2, \cdots\cdots, p)$ as linear combinations of type

$$\zeta_{\alpha j}=1_{\alpha 1}x_{1j}+1_{\alpha 2}x_{2j}+\cdots\cdots\cdots 1_{\alpha p}x_{pj} \quad \cdots\cdots\cdots(\text{II. }3)$$

and the number of the components ζ_α is equal to, or less than, p which is the number of the basal measures.

Those principal components have following properties.

$$\left.\begin{aligned} \frac{1}{n}\sum_j \zeta_{\alpha j}\zeta_{\beta j}&=0 \quad (\alpha\neq\beta) \\ &=\lambda_\alpha(\alpha=\beta) \end{aligned}\right\} \quad \cdots\cdots\cdots(\text{II. }4)$$

that is, those ζ's statistically uncorrelate and each variance of them is λ.

$$\sum_j \zeta_{\alpha j}=0,$$

for ζ is a linear combination of standardized values x_{ij}.

And as to coefficients 1's,

$$\left.\begin{aligned} \sum_i 1_{\alpha i}1_{\beta i}&=0 \quad (\alpha\neq\beta) \\ &=1 \quad (\alpha=\beta) \end{aligned}\right\} \quad \cdots\cdots\cdots(\text{II. }5)$$

If we regard n column vectors in matrix (II. 2) of x_{ij} as showing n ponits plotted on x-coordinates of p dimensions, then new matrix.

$$\begin{pmatrix} \zeta_{11}, & \zeta_{12}, & \cdots\cdots\cdots\zeta_{1n} \\ \zeta_{21}, & \zeta_{22}, & \cdots\cdots\cdots\zeta_{2n} \\ - & - & \cdots\cdots\cdots \\ \zeta_{p1}, & \zeta_{p2}, & \cdots\cdots\cdots\zeta_{pn} \end{pmatrix} \quad \cdots\cdots\cdots(\text{II. }6)$$

may be regarded as the same represented on ζ-coordinates. That is an orthogonal transformation or rotation in geometrical terminology. Therefore the vectors

$$(1_{\alpha 1}, \ 1_{\alpha 2}, \ \cdots\cdots\cdots 1_{\alpha p}) \quad \cdots\cdots\cdots(\text{II. }7)$$

are considered as the direction cosines of those new axes ζ's. Accordingly those ζ's are in the sense of the above mentioned geometrically orthogonal.

Usually we assign number α to each ζ_α in descending order of variance λ_α. Then λ_1, which is the variance of ζ_1, is not only larger than any other $\lambda_\alpha(\alpha\neq1)$, but the largest of variances along any other axes. That is, ζ_1 is showing the values on the axis along which the variance is the maximum of all obtained by orthogonal transformation of x-coordinates. And ζ_2 is the one having the largest variance in the space orthogonal to the axis ζ_1 and ζ_3

is the same in the space orthogonal to the axes ζ_1 and ζ_2, and so on.

2.2 An component analysis (No. 1)

Table (II. 2) is showing the values of the linear coefficients {1's} obtained by component analysis of the data used for the computation of table (II. 1) in section 1.

Table. II. 2. Values of the coefficients of each component

Basal measures	First component ζ_1	Second component ζ_2	Third component ζ_3	Fourth component ζ_4	Fifth component ζ_5
1. Mean gradient	0.41080 ◎	−0.13613	−0.15034	0.31438	0.80950 ◎
2. Density of all standing timber in number	−0.40006 ◎	−0.17945	−0.37225	0.07905	0.09892
3. Density of cut trees in number	−0.50394 ◎	0.06665	−0.23080	−0.21698	0.32138
4. Mean D. B. H.	0.05198	0.57256 ◎	0.08891	0.01401	0.12966
5. Variance of D. B. H.	0.35635 ○	0.24763	0.15224	−0.66540 ◎	0.19609
6. Mean height	0.06937	0.50806 ◎	−0.04882	0.60231 ◎	−0.12697
7. Density of cut trees in volume	−0.39111 ○	0.41074 ○	−0.12566	−0.11816	0.28156
8. Mean clear height	−0.28439	0.16109	0.63259 ◎	0.09997	0.10680
9. Mean number of branches	0.22112	0.32078	−0.57976 ◎	−0.13505	−0.26378
Variance λ of each ζ	3.05276	2.52172	1.32392	0.74323	0.49967
Proportion of λ	0.33920	0.28019	0.14710	0.08258	0.05552
Cumulative proportion of λ	0.33920	0.61939	0.76649	0.84907	0.90459
Notional meaning of each ζ	Standing timber distribution	Magnitude of cut trees	Tree shape	Forest type	Topography (gradient)

For instance, each value of first component ζ_1 is computed like this;

$$\zeta_{1j}=0.41080x_{1j}-0.40006x_{2j}-0.50394x_{3j}+0.05198x_{4j}+0.35635x_{5j}+0.06937x_{6j}$$
$$-0.39111x_{7j}-0.28439x_{8j}+0.22112x_{9j}$$
$$(j=1,\ 2,\ \cdots\cdots\cdots\cdots\cdots\cdots,\ n)$$

In table (II. 2) we marked ◎ on the coefficients of distinguished magnitude ○ on the fairly large ones.

Considering those coefficients are the direction cosines of ζ's, the following interpretations may be possible.

1) The first component ζ_1 ⋯⋯ General standing timber distribution

This component ζ_1 is composed of mainly five factors; mean gradient, density of all standing timber in number, density of cut trees in number, density of cut trees in volume, and variance of D.B.H. of cut trees.

Viewing over them excepting mean gradient, we may regard it as having represented general distribution of standing timber. The problem here is how to interpret the fact that the coefficient of mean gradient is large. Thinking that the fifth component purely represented gradient as will be mentioned afterwards and that the coefficient of mean gradient is small in the component representiong the same conception in next example, that may be regarded as expressing that the distribution of standing timber correlated with mean gradient there. In other words, it differed by steep or gentle of slope, and that aspect was a local characteristic of the area.

Accordingly we call this first component "general standing timber distribution" as the first important conceptual characteristic of the felling place. We consider that ζ_1 is an indicator of the distribution of standing timber of that area.

Observing the signs of those coefficients, we find they are positive for mean gradient and variance of D.B.H., and negative for other three kinds of densities. Therefore that the value of ζ_1 is small represents that those three kinds of densities were all thick, that the variance of D. B. H. was small, that is, trees were comperatively even in magnitude, that generally such a place was on the gentle slope. On the contrary, that ζ_1 is large expresses steep slope, thin in three kinds of densities, and uneven in magnitude of trees.

In fact, the actual place was the natural forest including Sugi and several kinds of broad leaved trees, and Sugi's were standing thickly on the ridges where the gradient was gentle, and there stood thin woods including a lot of broad leaved trees on the steep mountainsides.

2) The second component ζ_2 …… Magnitude of cut trees

This component ζ_2 is composed of mainly mean D.B.H., mean height and density of cut trees in volume. The coefficient of the last is a little smaller than the other two.

Excepting density in volume, as the signs of the coefficients are the same, this may be regarded as representing magnitude of cut trees in average. Considering that volume of standing tree is computed from its D.B.H. and height and that the coefficient of density of cut trees in number is very small, that is, this component practically uncorrelates with it, it is a matter of course that the coefficient of density in volume is a little distinctive.

From the above mentioned facts, we can understand the second component ζ_2 having represented the conception of magnitude of cut trees.

Here comparing those coefficients with the ones of the first component, we find those of mean D.B.H. and mean height which are main members of ζ_2 are negligible in ζ_1. The fact suggests that the magnitude of cut trees was strongly independent of the distribution of standing timber there.

3) The third component ζ_3 …… Tree shape

We will regard this third component ζ_3 as representing the shape of cut trees, because it is mainly composed of mean clear height and mean number of branches and it is possible to call the charactor expressed by them the shape of trees.

The larger the value of ζ_3 was, the higher the clear height and the fewer the number of branches were. The coefficients of mean D.B.H. and mean height are negligible, so we can say that the shape of trees did not correlate with the magnitude of cut trees in the area.

4) The fourth component ζ_4 …… Forest type

This is mainly composed of variance of D.B.H. of cut trees and mean height, whose signs of coefficients are negative for the former and positive for the latter. So, the larger the value of ζ_4 was, the smaller the variance of D.B.H. and the higher the mean height was, and vice versa.

In case of the well matured forest, we can think that the forest of which the mean height is high and the D.B.H.'s are comparatively uniform is of regular type and that it is of irregular type in the contrary case. Therefore we may regard the component ζ_4 as representing the conception of the type of forest, and that is indicating the change level between of regular type and of irregular type.

5) The fifth component ζ_5 …… Mean gradient (Topography)

Although the variance of this component is fairly small, 0.4997, it is very distinctive that this is represented by almost only mean gradient. Therefore we will take it account this ζ_5 as representing the conception of topography or just mean gradient.

As to the sixth to nineth components, those variances λ_6–λ_9 were all small, and any significant interpretations of them could not be obtained. And the sum of variances of 1st to 5 th components has already been over 90 % of total variance, so we can say that the above five components have already covered almost all the variety of the felling place originally expressed by nine basal measures. In geometrical words, those points originally represented on x-coordinates of nine dimensions could be almost sufficiently represented on ζ-coordinates of only five dimensions.

The conclusion suggests that each component which is statistically independent with one another represented each of five conceptual characteristics as follows: standing timber distribution, magnitude of cut trees, tree shape, and forest type, and topography.

If the above mentioned matters are true in general, the conception of principal components will be very useful for representing the variation of the felling place, for they do not correlate one another and they represent each different characteristic of the places.

In the next paragraph, we will analyze another example to confirm whether the above outcomes are by chance or not.

2.3 An component analysis (No. 2)

The data used for this example were also obtained in the natural forest which was including Sugi and several broad leaved trees in Kyoto University Forest at Ashu, 1961. The data are quite different from those used in the former paragraph 2.2.

The following six measures were observed at each of sixteen plots, which was about 200–400 squaremeters in area including ten cut trees (Sugi); mean gradient, density of cut trees in number, mean D.B.H. of cut trees, variance of D.B.H., mean height, and density of cut trees in volume.

Table II.3 is showing the correlation matrix computed from the data.
And the coefficients of principal components computed from matrix II. 3 are shown on table II. 4.

As before, we marked ◎ on the coefficients which were very remarkable, and ○ on those which were fairly large. Then we can interpret each component observing those marks.

1) The first component …… Magnitude of cut trees

As the coefficients of mean D.B.H. and mean height are remarkable and have the same sign, we can regard the component as representing the magnitude of cut trees. And this

Table. II. 3. Correlation matrix of basal measures (Number of data: 16)

	1. Mean gradient	2. Density of cut trees in number	3. Mean D. B. H.	4. Varience of D. B. H.	5. Mean height	6. Density of cut trees in volume
1. Mean gradient	1.0000	−0.0468	0.1185	−0.0404	0.3385	−0.0375
2. Density of cut trees in number	−0.0468	1.0000	−0.4518	0.1135	−0.4678	0.6555
3. Mean D. B. H.	0.1185	−0.4518	1.0000	0.5349	0.8118	0.3375
4. Variance of D. B. H.	−0.0404	0.1135	0.5349	1.0000	0.2191	0.6508
5. Mean height	0.3385	−0.4678	0.8118	0.2191	1.0000	0.1986
6. Density of cut trees in volume	−0.0375	0.6555	0.3375	0.6508	0.1986	1.0000

Table. II. 4. Values of the cofficients of each component

Basal measures	First component ζ_1	Second component ζ_2	Third component ζ_3	Fourth component ζ_4
1. Mean gradient	0.1619	−0.1691	0.9020 ◎	−0.3306
2. Density of cut trees in number	−0.1935	0.3096 ◎	0.3030	0.2561
3. Mean D. B. H.	0.6040 ◎	−0.1052	−0.1649	0.0867
4. Variance of D. B. H.	0.4236 ○	0.3741 ○	−0.2056	−0.7092 ◎
5. Mean height	0.5429 ◎	−0.2455	0.1320	0.4798 ○
6. Density of cut trees in volume	0.3121	0.6017 ◎	0.0887	0.2904
Variance λ of each ζ	2.4558	1.9624	1.0177	0.4602
Proportion of λ	0.4093	0.3207	0.1696	0.0767
Cumulative proportion of λ	0.4093	0.7300	0.8969	0.9763
Notional meaning of each ζ	Magnitude of cut trees	Standing timber distribution	Topography (gradient)	Forest type

should be identified with the second component of the precedent example.

2) The second component …… Standing timber distribution

This is mainly composed of density of cut trees in volume and density of cut trees in number. So we can regard this as representing the conception of the distribution of standing timber. Comparing it with the first component of the precedent example, we can see there the same tendency in signs, but this has not such a tight correlation with gradient as before. Because of the fact, we concluded in the precedent example that gradient did not play a basal role for the meaning of the component, and that it was only expressing a local feature of the characteristic "standing timber distribution."

3) The third component …… Mean gradient (Topography)

As this component is expressed by almost only mean gradient, we can regard it as representing topography or only mean gradient of the area like the fifth component in the precedent example.

4) The fourth component …… Forest type

This is mainly composed of variance of D.B.H. and mean height, and their signs are the same as the precedent fourth component, that is, negative for the former and positive for the latter. Therefore we can regard it as also representing the conception of forest type, which indicates a change level between of regular and of irregular type as before.

The sum of variance of the above four components has already been 98 % of total, so these four components can be considered to represent almost all the variations caught by the six basal measures.

Thus the characteristics interpreted in two examples coincided very well with each other. Only there did not include such a component as represents tree shape in this example, because any basal measures relating to it were not picked up.

2. 4 An expression of the variety of the place of felling by characteristic components

As explained in two examples, it is, to some extent, possible to transform collectively the variety of the felling place into some independent components. We will call them "characteristic components" to the effect that they represent the characteristics of the felling place, and we can use them as some quantitative measures of the variety of the felling place.

We can pick up and discuss about each characteristic component separately, because of their statistical independence as mentioned before two or three times. Therefore even when we discuss about the relation with the working rate or about the changes of some characteristics of the felling place itself, we do not have such a trouble as when we use basal measures. Then we can classify the felling places making use of those characteristic components. And we can study resemblances and differences of different places by comparing the values ζ_i's.

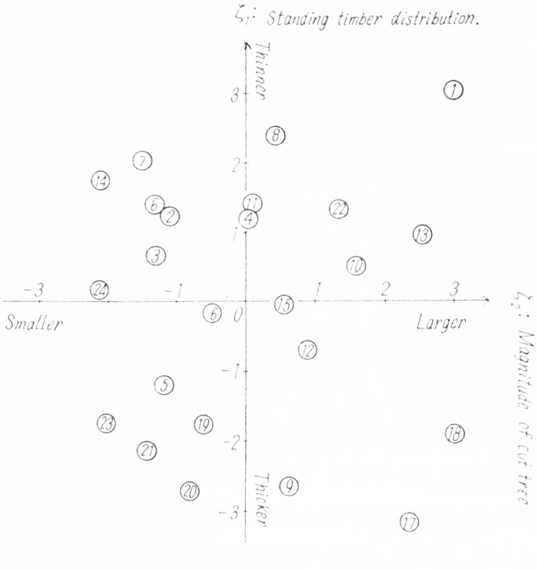

Fig. II. 1. Characteristic Value of Each Plot. (No. 1)

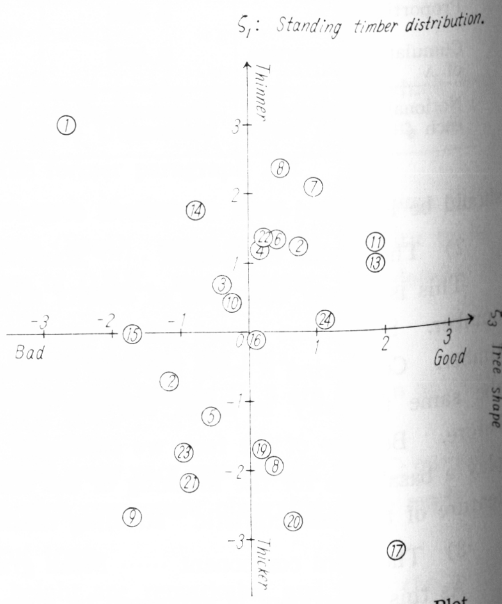

Fig. II. 2. Chracteristic Value of Each Plot. (No. 2)

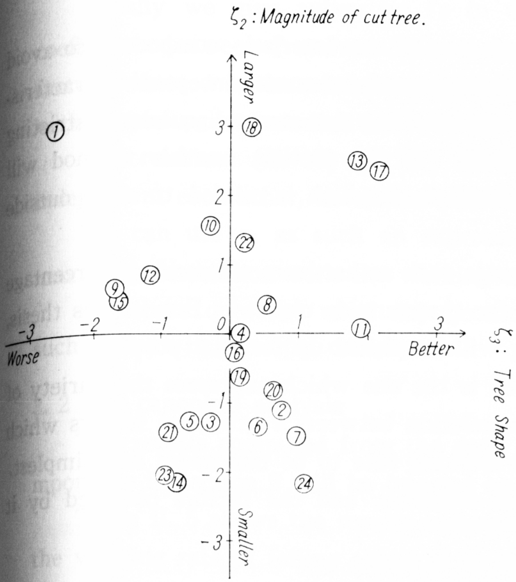

Fig. II. 3. Characteristic Vatue of Each Plot. (No. 3)

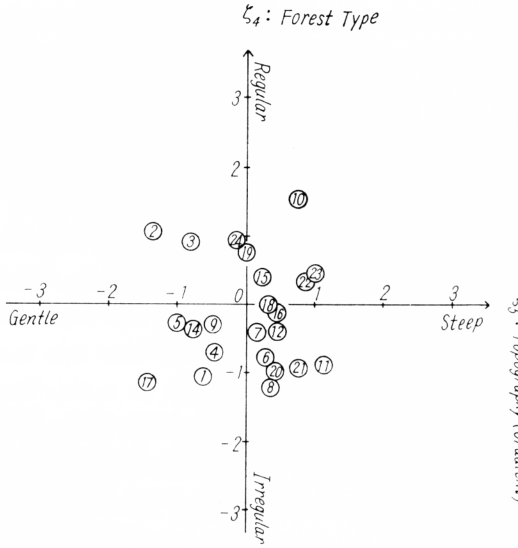

Fig. II. 4. Characteristic Value of Each Plot. (No. 4)

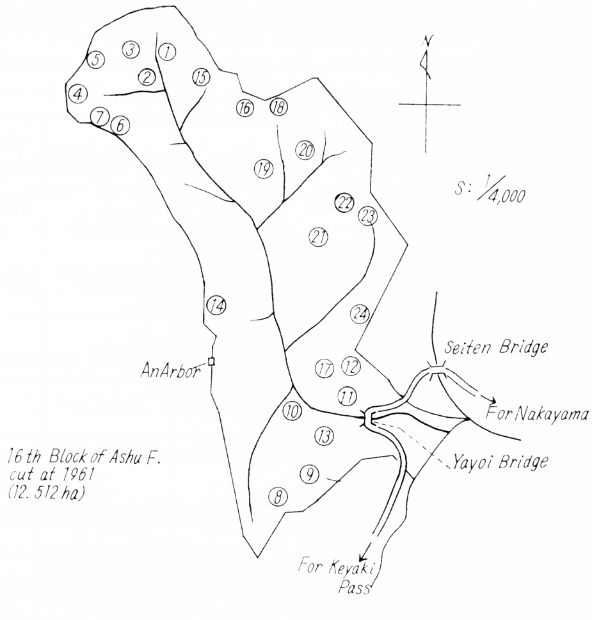

Fig. II. 5. Location of Each Plot.

Here we will plot the values of 24 investigated plots in the first example on ζ-coordinates. In that example, the variety of the felling place was represented by five characteristic components which are geometrically orthogonal, so those 24 points must be plotted on the space of five dimensions. But it is impossible to write them on flat paper, so we divide them into four graphs of two dimensions each. Figures II. 1~II. 4 are those.

Figure II. 5 shows actual location of each investigated plot.

3. Canonical factors

In the former section, an expression technique named characteristic component to avoid unconvinience of basal measures by correlation was suggested and some conceptual characteristics of the felling place were embossed in two examples. But that was a discussion restricting our sight within the inner part of the felling place. In this section, another method will be proposed for expressing the variety of the felling place which is related to some outside objects.

Any factor which is expressed by digital figures such as hardness of working, percentage of some specified elementary work, etc. will do for that outside object. But in this thesis, only the working rate of felling and log-making will be picked up for that purpose.

Flatly speaking, we can say that this method is the one which expresses the variety of the felling place in such a way as makes the correlation between a group of factors which represent it and another group representing the working rate to be maximum and simplest. This technique is so called canonical analysis in statistics, and the factors obtained by it will be called canonical factors tentatively.

3. 1 Some properties of canonical factors

A lot of books on canonical analysis have already been published, but at least principal properties of canonical factors should be explained here for the future discussions.

Suppose standardized values of p variables representing the working rate of felling are

$$y_1,\ y_2,\ \cdots\cdots\cdots,\ y_p,$$

and canonical factors (p in number) computed from y_i's are

$$\zeta_1,\ \zeta_2,\ \cdots\cdots\cdots,\ \zeta_p.$$

And suppose standardized values of q basal measures representing the variety of the felling place are

$$x_1,\ x_2,\ \cdots\cdots\cdots,\ x_q,$$

(for convenience $p \leqq q$), and those canonical factors (p in number) are

$$\eta_1,\ \eta_2,\ \cdots\cdots\cdots,\ \eta_p.$$

Then each canonical factor is a linear combination of basal measures of type

$$\zeta_i=\alpha_{i1}y_1+\alpha_{i2}y_2+\cdots\cdots+\alpha_{ip}y_p \quad \cdots\cdots\cdots(\text{II. } 8)$$

$$\eta_i=\beta_{i1}x_1+\beta_{i2}x_2+\cdots\cdots+\beta_{ip}x_p \quad \cdots\cdots\cdots(\text{II. } 9)$$

$$(i=1,\ 2,\ \cdots\cdots,\ p).$$

And those η_i's are canonical factors which represent the variety of the felling place as a whole.

Properties of canonical factors ζ_i, $\eta_i (i=1,\ 2,\ \cdots\cdots,\ p)$ are as follows:

$$\left.\begin{aligned}\frac{1}{n}\sum(\zeta_i\zeta_j)&=0 \quad (i\neq j)\\ &=1 \quad (i=j)\end{aligned}\right\} \quad \cdots\cdots\cdots(\text{II. } 10),$$

$$\left.\begin{aligned}\frac{1}{n}\sum\eta_i\eta_j&=0 \quad (i\neq j)\\ &=1 \quad (i=j)\end{aligned}\right\} \quad \cdots\cdots\cdots(\text{II. } 11),$$

$$\left.\begin{aligned}\frac{1}{n}\sum\zeta_i\eta_j&=0 \quad (i\neq j)\\ &=\lambda_i \quad (i=j)\end{aligned}\right\} \quad \cdots\cdots\cdots(\text{II. } 11).$$

(n: number of data)

Usually we give subscripts i's in descending order of magnitude of the correlation coefficients between ζ and η. Then λ_1 is the maximum of them and ζ_2, η_2 are the canonical factors whose λ is the maximum of those which satisfy the relations (II. 10) (II. 11) (II. 12) with ζ_1 and η_1, and similar for $i=3, 4, \cdots\cdots\cdots, p$.

The relations (II. 10) (II. 11) (II. 12) indicate that canonical factors are statistically independent each other, no matter whose expression they may be.

We can use η_i as such an expression of the variety of the felling place as having maximum correlation with the working rate.

But there does not exist any geometrical orthogonality among those canonical factors such as seen in principal components.

3. 2 An canonical analysis

The results computed from the data which were used for computing characteristic components in section 2 will be inserted as an example of canonical factors.

Table II. 5 shows the results computed from the same data that in paragraph 2. 2 and the working rate of felling observed in the same places.

Figures in table II. 5 are the values of α_{ij} and β_{ij} which are coefficients of linear combinations. For example,

$$\zeta_1=0.99176y_1+0.26652y_2$$

$$\eta_1=-0.43581x_1-0.03074x_2-\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots+0.04182x_9.$$

Table. II. 5. Coefficients of canonical factors (for the rate of felling work)

Factors expressing the rate of felling work	ζ_1	ζ_2
1. The amount of work per actual working man-hour	0. 99176	−0. 16502
2. The actual working time rate	0. 26652	0. 96939
Basal measures	η_1	η_2
1. Mean gradient	−0. 43581	−0. 41480
2. Density of all standing timber in number	−0. 03074	0. 09985
3. Density of cut trees in number	−1. 06658	0. 71208
4. Mean D. B. H.	−0. 64298	0. 92554
5. Variance of D. B. H.	0. 12160	0. 16747
6. Mean height	0. 16393	−0. 18065
7. Density of cut trees in volume	1. 44701	−1. 59227
8. Mean clear height	0. 41783	0. 20476
9. Mean number of branches	0. 04182	−0. 62469
Correlation coefficient of each pair $\zeta:\eta$	0. 8760	0. 5042

Table. II. 6. Coefficients of canonical factors (for the rate of log-making work)

Factors expressing the rate of log-making work	ζ_1	ζ_2
1. The amount of work per actual working man-hour	0. 3688	0. 9344
2. The actual working time rate	0. 9657	−0. 2775
Basal measures	η_1	η_2
1. Mean gradient	−0. 1337	0. 0003
2. Density of cut trees in number	−2. 9841	−3. 3833
3. Mean D. B. H.	−1. 8698	−0. 4212
4. Variance of D. B. H.	−0. 5538	−1. 4393
5. Mean height	−0. 9782	−1. 0952
6. Density of cut trees in volume	3. 5051	3. 6350
Correlation coefficient of each pair $\zeta:\eta$	0. 8864	0. 6395

We used

y_1: the amount of work in volume per actual working man hour, and

$$y_2\text{: actual working time rate} = \frac{\text{total of actual working hours}}{\text{total of working hours}} \times 100$$

as basal factors for representing the working rate of felling.

Table II. 6 shows the results computed from the same data that in paragraph 2. 3 and the working rate of log-making observed there.

Observing those results of canonical analysis, we understand those canonical factors η which represent the variety of the felling place can not be interpreted in such a way as done in the case of characteristic components. But as for canonical factors ζ which represent the working rate, we obviously be able to interpret as follows.

To describe about the results in table II. 5, ζ_1 represents almost only the amount of work in volume per actual working time y_1, and ζ_2 the actual working time rate y_2. The amount of work and actual working time rate are practically uncorrelated each other. Consequently η_1 is such a measure of the variety of the felling place as having maximum correlation with the amount of work, and η_2 is the same with actual workig time rate.

And describing adout table II. 6, η_1 and η_2 are measures of it which have the most tight correlation with actual working time rate and with the amount of log-making, respectively.

III. Influence of the variety of the place of felling on the working rate

In chapter II, two statistical methods were proposed for representing the variety of the felling place. One is characteristic components and the other is canonical factors. Then let's discuss the problem how the variety of the felling place caught by those two methods influences on the working rate of felling and log-making analyzing the precedent examples.

1. Analysis by characteristic components

Now back to the precedent examples, which are investigated at Ashu. There the following working order was adopted;

$$\underbrace{\text{felling} \cdots\cdots \text{branch cutting} \cdots\cdots \text{barking}}_{\text{felling process}} \cdots\cdots \text{seasoning} \cdots\cdots \underbrace{\text{log cutting}}_{\text{log-making process.}}$$

Consequently, the working order was divided into two parts by seasoning process. And the felling process including felling, branch cutting, and barking before seasoning and the log-making process after it are independent each other, so we can deal with them separately.

We observed the working rate of felling on 24 plots of which the variety was analyzed in II. 2. 2, and that of log-making on 16 plots of which the variety was analyzed in II. 2. 3.

1. 1 An analysis on the working rate of felling process.

In paragraph II. 2. 2, we transformed the variety of 24 places into five conceptual characteristics: standing timber distribution ζ_1, magnitude of cut trees ζ_2, tree shape ζ_3, forest type ζ_4, and mean gradient ζ_5.

We represented the working rate of felling process by the amount of felled trees in volume per actual working man hour and the actual working time rate.

Any characteristic component ζ is zero sum and statistically independent, so we can deal with each ζ as a orthogonal comparison and do partition of sum of squares.

The results of analysis of variance of each factor of the working rate by those five components are table III. 1 and III.2.

Table. III. 1. The table of analysis of variance by each characteristic factor about the amount of felling work in volume per actual working man-hour.

Factors	d. f.	S. S.	M. S.	F^{1}_{18}
ζ_1 : Standing timber distribution	1	0.16587	0.16587	7.8798*
ζ_2 : Magnitude of cut trees	1	0.37175	0.37175	17.6603**
ζ_3 : Tree shape	1	0.04946	0.04946	2.3496
ζ_4 : Forest type	1	0.00000	0.00000	0.0000
ζ_5 : Topography	1	0.03404	0.03404	1.6171
Error	18	0.37888	0.02105	—
Total	23	1.00000	—	—

Table. III. 2. The table of analysis of variance by each characteristic factor about the actual working time rate of felling.

Factors	d. f.	S. S.	M. S.	F^{1}_{18}
ζ_1 : Standing timber distribution	1	0.01267	0.01267	0.2976
ζ_2 : Magnitude of cut trees	1	0.01155	0.01155	0.2713
ζ_3 : Tree shape	1	0.19215	0.19215	4.5137*
ζ_4 : Forest type	1	0.00764	0.00764	0.1794
ζ_5 : Topography	1	0.00973	0.00973	0.2285
Error	18	0.76626	0.04257	—
Total	23	1.00000	—	—

A star * in the tables means significance on 5 % level and two star mark ** means the same on 1 % level.

From table III. 1 which shows the result on the amount of felled trees per actual working man hour, standing timber distribution ζ_1 and magnitude of cut trees ζ_2 are significant. Let y_1 denote the amount of felled trees in volume per actual workig man hour, then the following relation is formed by least square method;

$$y_1 = -0.23310\zeta_1 + 0.38396\zeta_2 + Us \quad \cdots\cdots\cdots\cdots\cdots\cdots (\text{III. } 1)$$

$$s^2 = 0.46238, \quad U \varepsilon N(0,\ 1)$$

Here, y_1 is standardized and dimensionless, and $U \varepsilon N\ (0,\ 1)$ means that U is subject to the standard normal distribution.

Figures (III. 1) and (III. 2) show the relation (III. 1) on graphs.

From table III. 2 which shows the actual working time rate, only tree shape ζ_3 is significant, and denoting actual working time rate by y_2, we get the relation

$$y_2=0.38097\zeta_3+Us\cdots\cdots(\text{III. } 2).$$
$$s^2=0.80785,\quad U\varepsilon N\ (0,\ 1)$$

Figure III. 3 shows the relation (III. 2).

i) Influence on the amount of felled trees in volume per actual working man hour

As ζ_1 and ζ_2 in relation (III. 1) are statistically independent each other, we can discuss about them separately.

α) Influence of standing timber distribution ζ_1

From the discussion in paragraph II. 2. 2 and table II. 2, when the value of ζ_1 is large, it indicates density of all trees in number, density of cut trees in number and density of cut trees in volume were all thin and variance of D.B.H. of cut trees was large and the slope was steep there. When ζ_1 is small on the contrary, it shows that the slope was gentle, that three densities were thick, and trees were comparatively uniform in D.B.H..

Relation (III. 1) shows the amount of work was low in such a place as ζ_1 was large and high in case of the contrary, for the coefficient of ζ_1 is negative. That is, the amount of work was low in comparatively thin woods and high in thick woods.

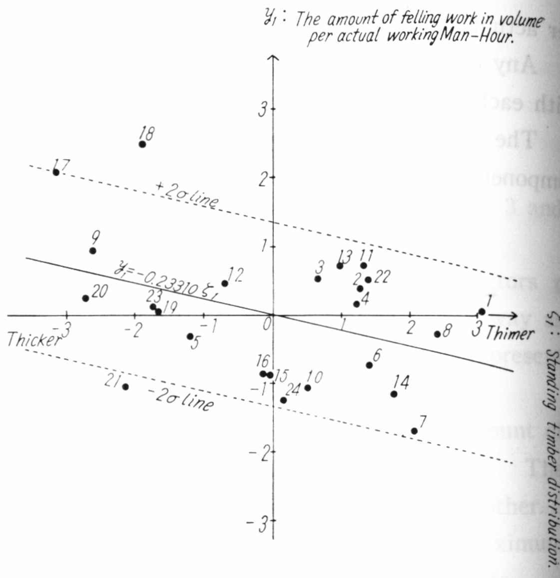

Fig. III. 1. The amount of felling work and standing timber distribution.

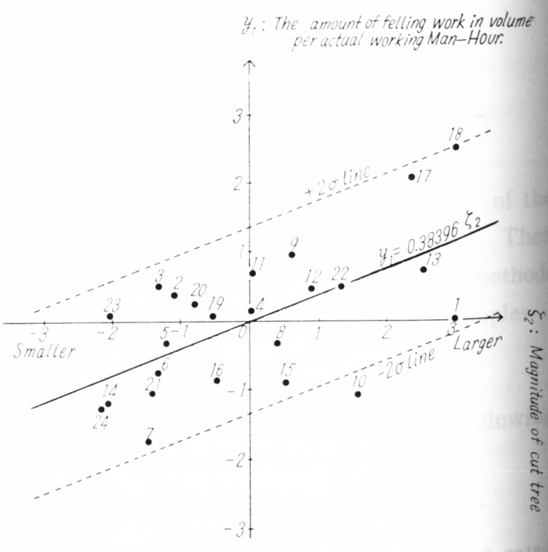

Fig. III. 2. The amount of felling work and magnitude of cut tree.

The coefficient of mean gradient was remarkable in ζ_1. As discussed in II. 2. 3, that should be thought to express a local feature of distribution of standing timber, so we should think that mean gradient did not have any essential influence on the working rate. That is confirmed by the fact that ζ_5 representing mean gradient itself did not have any significant influence on it.

β) Influence of magnitude of cut trees ζ_2

Large value of ζ_2 indicates that the cut trees were comparativly large in average and vice versa. So relation (III. 1) shows the proportional relation that the amount of felling work was much when the size of cut trees was large and little when it was small.

Above is how each characteristic component influenced on the amount of felled trees per actual working man hour in the example. Then how much was the influence power of each?

Each value in column S.S. in table III. 1 is variance of each orthogonal comparison which represents share of each characteristic component in total variance. So, we can regard each value of S.S. as indicating each influence power. We will call each of them contribution rate dividing by total S.S..

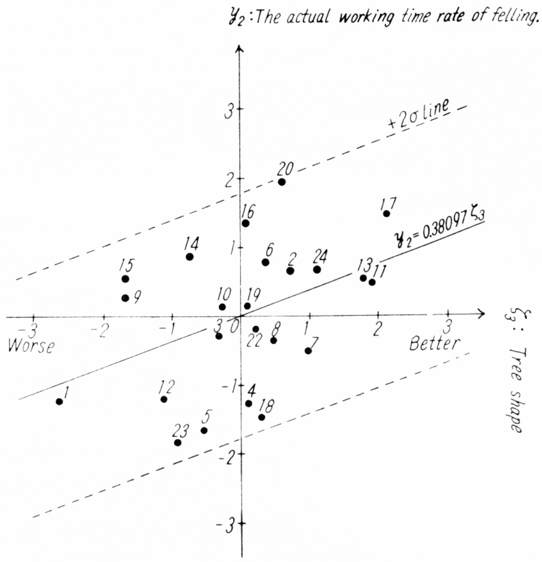

Fig. III. 3. The actual working time rate of felling and tree shape.

Then the contribution rate of ζ_1 is about 16.6 % and ζ_2's about 37.2 % from table III. 1. Accordingly, the conception magnitude of cut trees had the greatest influence on the amount of felling work. Summing up those two, it becomes about 54 %, so we can say that the greater part of the variance of the amount of felling work in the example was due to those two characteristics of the felling place.

ii) Influence on the actual working time rate of felling

From table III. 2, only ζ_3 which expresses tree shape had significant influence on the actual working time rate of felling. And the contribution rate of ζ_3 is about 19 %.

From table II. 2, large value of ζ_3 indicates mean clear height was comparatively high and number of branches was few and vice versa. As the coefficient of ζ_3 in relation (III. 2) is positive, we can understand the actual workig time rate was high in such places as mean clear height was high and number of branches few. And it was low in the places where mean clear height was low and number of branches many.

Those results may be thought to be a little strange, but those could come out if it was true that the resting time rate of branch cutting and barking was larger than that of felling, because it would need comparatively longer time for branch cutting and barking in the places where ζ_3 was small and shorter time when it was large.

1. 2 An analysis on the working rate of log-making process

We represented the working rate of log-making by the amount of logs in volume per actual working man hour and the actual working time rate of log-making process. And we performed analysis of variance of each of them 'by four characteristic components obtained in paragraph II. 2. 3: magnitude of cut trees ζ_1, standing timber distribution ζ_2, mean gradient ζ_3, and forest type ζ_4. Tables III. 3 and III. 4 are the results.

Table. III. 3. The table of analysis of variance by each characteristic factor about the amount of log-making work in volume per actual working man-hour.

Factors	d. f.	S. S.	M. S.	F^1_{11}
ζ_1 : Magnitude of cut trees	1	0. 05059	0. 05059	0. 7851
ζ_2 : Standing timber distribution	1	0. 00026	0. 00026	0. 0040
ζ_3 : Topography	1	0. 16201	0. 16201	2. 5145
ζ_4 : Forest type	1	0. 07837	0. 07837	1. 2164
Error	11	0. 70877	0. 06443	—
Total	15	1. 00000	—	—

Table. III. 4. The table of analysis of variance by each characteristic factor about the actual working time rate of log-making.

Factors	d. f.	S. S.	M. S.	F^1_{11}
ζ_1 : Manitude of cut trees	1	0. 1921	0. 1921	5. 6334*
ζ_2 : Standing timber distribution	1	0. 3718	0. 3718	10. 9032**
ζ_3 : Topography	1	0. 0557	0. 0557	1. 6334
ζ_4 : Forest type	1	0. 0049	0. 0049	0. 1436
Error	11	0. 3755	0. 0341	—
Total	15	1. 0000	—	—

i) Influence on the amount of logs in volume made per actual workig man hour

None of those characteristic components had any significant influence on the amount of log-making work, as shown in table III. 3. But we can not think the variety of the place did not have any influence on it. That is the conclusion only relative to four characteristics which were analyzed actually.

ii) Influence on the actual working time rate of log-making

We can see on table III. 4 that ζ_1 representing magnitude of cut trees and ζ_2 standing timber distribution were significant for the actual working time rate of log-making.

Let y_3 denote standardized value of the actual working time rate of log-making, then

$$y_3 = -0.2797\zeta_1 + 0.4353\zeta_2 + Us \quad \cdots\cdots \text{(III. 3)}.$$

$$s_2 = 0.4361, \quad U \varepsilon N\ (0,\ 1)$$

We can understand from table II. 4 and relation (III. 3) that the actual working time rate was comparatively low in such a place as magnitude of cut trees ζ_1 was large and that when ζ_1 was small it was high. And from the second term of (III. 3), the actual working time rate was high in thick woods and low in thin woods, because large value of ζ_2 indicates that densities of cut trees in number and in volume were both thick, and the small value shows those densities were thin.

From S.S. column of table III. 4, the contribution rates of ζ_1 and ζ_2 to the actual working time rate of log-making are 19.2 % and 37.2 % respectively.

In case of the felling process, such charactristics did not have significant influence on the actual working time rate, but in this log-making case they had decisive influence, especially thick or thin of standing timber distribution had quite large influence on it.

2. Analysis by canonical factors

2. 1 An analysis on the working rate of felling process

In section II. 3, we have already obtained canonical factors ζ_1 and ζ_2 which represent the working rate and η_1 and η_2 representing the variety of the felling place of the first example. And they all uncorrelate one another except two pairs $(\zeta_1\eta_1)$ and $(\zeta_2\eta_2)$, of which correlation coefficients are λ_1 and λ_2 respectively. So we can discuss about each pair separately.

We consider about $(\zeta_1\eta_1)$ first. From table II. 5,

$$\zeta_1=0.99176y_1+0.26652y_2,$$

so ζ_1 represents mainly y_1.

We now derive the regression function of ζ_1 and η_1.

Suppose the regression function is of type

$$\zeta_1=\alpha_1\eta_1+Us \qquad \text{(III. 4)},$$

$$U\varepsilon N(0,\ 1)$$

from the fact that each variance of ζ_1 and η_1 is unit, immidiately

$$\alpha_1=\frac{E(\zeta_1\eta_1)}{E(\eta_1^2)}=\lambda_1 \qquad \text{(III. 5)}$$

$$s^2=E(\zeta_1-\alpha_1\eta_1)^2=1-\lambda_1^2 \qquad \text{(III. 6)},$$

Here E means "expectation". And we can do test of significance of coefficient α_1 making use of the fact that, provided n is number of data,

$$T_1=\frac{\lambda_1}{\sqrt{\dfrac{s^2}{n-1}}} \qquad \text{(III. 7)}$$

is subject to t-distribution of $(n-1)$ degree of freedom.

Accordingly,

$$\zeta_1=0.8760\,\eta_1+Us \qquad \text{(III. 8)}$$

$$s^2=0.2326$$

$T_1=8.7164$ (d. f. 23: significant on 1% level).

Figure III. 4 shows the relation.

Similarly the regression function of ζ_2 on η_2 is derived like

$$\zeta_2=0.5042\,\eta_2+Us \qquad \text{(III. 9)}$$

$$s^2=0.7458$$

$T_2=2.8011$ (d. f. 23: significant on 5% level).

And figure III. 5 shows the relation.

2. 2 An analysis on the working rate of log-making process

We have got pairs $(\zeta_1\eta_1)$ and $(\zeta_2\eta_2)$ of the second example in paragraph II. 3. 2, and we can immidiately derive the following regression functions;

$$\zeta_1=0.8864\,\eta_1+Us \qquad \text{(III. 10)}$$

$$s^2=0.2143$$

$T_1=7.4176$ (d. f. 15: significant on 1% level),

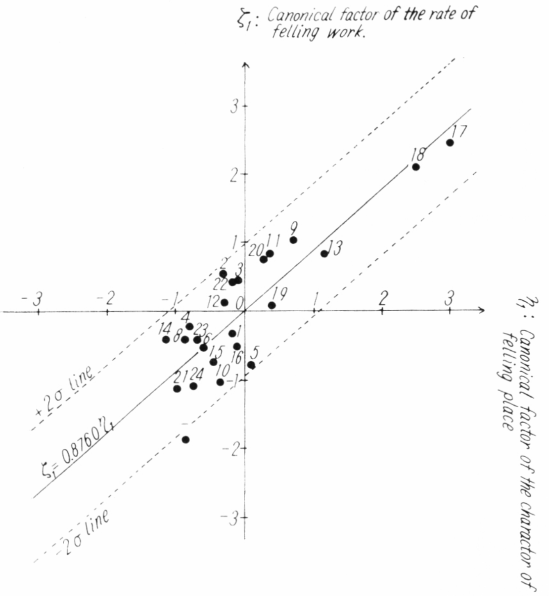

Fig. III. 4. The relation between the rate of felling work and the charactor of felling place by canonical factor.

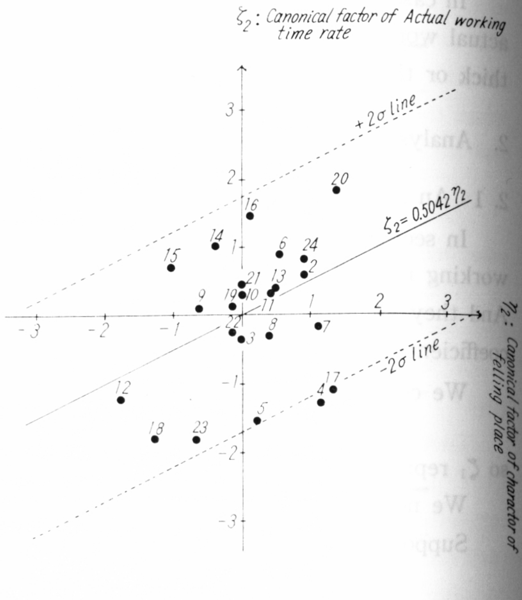

Fig. III. 5. The relation between the actual working time rate of felling and the charactor of forest by canonical factor.

and $\zeta_2 = 0.6395\,\eta_2 + Us$ ....(III. 11)

$s^2 = 0.5910$

$T_2 = 3.2216$ (d. f. 15: significant on 1 % level).

IV. Conclusion

The author suggested two statistical techniques for representing the variety of the felling place: one is component analysis and the other is canonical analysis, and surveyed the relation between the working rate of felling and of log-making and the variety of the felling place by two actual cases.

By component analysis, the variety of the felling place was transformed into five characteristic components equally in any of two cases, each of which expressed each of the conceptions; standing timber distribution, magnitude of cut trees, tree shape, forest type, and mean gradient.

The following results were obtained about influence of those characteristics on the working rate.

i) On the working rate of felling process, the amount of felled trees in volume per actual working man hour had some relations with standing timber distribution and magnitude of cut trees; it increased with the thickness and magnitude of cut trees and decreased with the contraries. And the actual working time rate of felling work was influenced by tree shape; it was high when the tree shape was good, that is, in case of clear height was high and number of branches was few, and low in the contrary case.

ii) On the working rate of log-making, the amount of logs in volume made per actual working man hour was not recognized to have any significant relation with those characteristics, but the actual working time rate of log-making had some relations with magnitude of cut trees and standing timber distribution; it was lowered with magnitude of cut trees and thinness of the standings and vice versa.

Those results are all obtained about two cases observed in a part of Kyoto University Forest at Ashu. But the author thinks we may expect that the fact that we could extract five characteristics which are statistically independent and those tendencies of influence on the working rate will be verified in more general cases.

On the other hand, the author does not think we succeeded in the method of canonical analysis like the former, as far as those two cases are concerned. Although it creates the simplest and maximum relations between the working rate and the variety of the felling place in mathematical sense, we can not neglect there is remained something obscure and it is a little troublesome to treat with.

But anyway those methodologies must be investigated and developed in the future, and they will be powerful for our purpose some day.

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伐採地の性格とその作業功程への影響

神　崎　康　一

要　旨

筆者は森林の多様性を表現するために二つの統計的方法を用いることを提案し，実際の伐採作業の二つの場合について，その作業功程と作業地の多様性との関係を解析してみた。その二つの統計的方法とは，Component Analysis 法と Canonical Analysis 法のことである。

作業地の多様な様相は，Component Analysis により，二つの例において，全く同様な五つの統計的に独立な概念的特性の型にまとめることが出来た。その五つの概念的特性とは，立木分布状態，伐採木の大きさ，樹形，林形，平均勾配である。そうして，これらの特性の作業功程への影響として，次のような結果が得られた。

i）伐木作業について

この伐木作業とは，天然スギの伐倒・枝払・剥皮を一まとめにしたものであるが，この作業の実働1時間1人当り出来高が，立木分布状態と伐採木の大きさによって影響を受けた。この実働時間当り出来高は，立木の分布が密である程，また，伐採木が平均的に大きい程，多くなり，一方，その反対の場合に少くなるという傾向があった。

また，この伐木作業の実働率は，樹形といくらかの関連があり，樹形の良い場合，すなわち，枝下が高く，枝数の少いとき高率を示し，その逆の場合には低くなった。

ii）玉切作業の場合

上に述べた五つの特性は，いずれも玉切作業の実働1時間1人当り出来高に対して有意な影響が見られなかった。しかし，この実働率は，立木分布状態及び伐採木の大きさと関連があり，立木分布が密である程，また，伐採木の平均的大きさが小さい程，高率であり，その逆の場合に低くなる傾向があった。

以上の結果は，いずれも京都大学芦生演習林の一部で行われた作業について出て来たものであるが，筆者は，上のような結果，特に伐採地の多様性がいくつかの概念的特性によって把へられるという結果は，もっと一般的に成立しうるのではないかと考える。

一方，Canonical Analysis による分析は，森林の多様性と作業功程との関係を，もっとも数学的に簡単で，かつもっとも強い相関をもつ型で，取り出してくれるが，前の方法のような意味づけが困難なので，いくぶん，この例の場合，取扱いに面倒なように思われた。